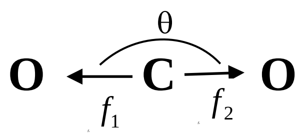
In this article we would like to prove mathematically that the angle H-C-H in methane structure is 109.5°.

Let's first recall dot product:

$$a \bullet b = |a||b|\cos\theta \text{ i.e. } a \bullet a = |a||a|\cos 0 = |a|^2$$

As a warm up let's consider CO_2. This a linear structure and hence the angle O = C = O is clearly 180°.
Let's use dot product to prove this:



Consider forces on the carbon atom we know:

$$f_1 + f_2 = \mathbf{0} \text{ and } |f_1| = |f_2|$$

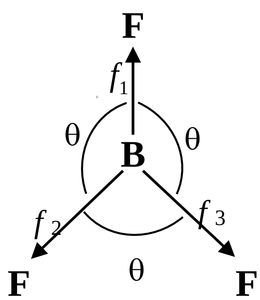
$$f_1 \bullet (f_1 + f_2) = f_1 \bullet \mathbf{0}$$

$$|f_1|^2 + |f_1||f_2|\cos\theta = 0$$

$$\cos\theta = -1$$

$$\theta = 180^0$$

Next let's consider BF_3. This a planar structure and hence the angle F – B - F is clearly 120°.
Let's use dot product to prove this:



Consider forces on the Boron atom we know:

$$f_1 + f_2 + f_3 = \mathbf{0} \text{ and } |f_1| = |f_2| = |f_3|$$

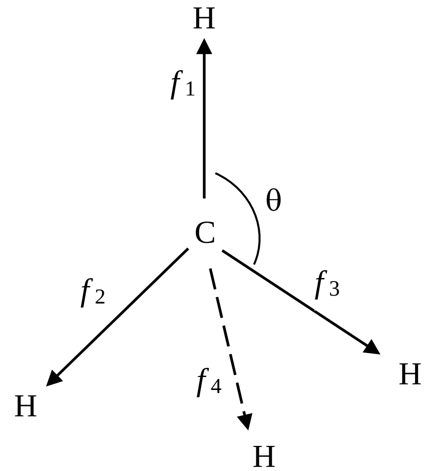
$$f_1 \bullet (f_1 + f_2 + f_3) = f_1 \bullet \mathbf{0}$$

$$|f_1|^2 + |f_1||f_2|\cos\theta + |f_1||f_3|\cos\theta = 0$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120^0$$

Finally let's consider CH_3. This a 3-Dimensional structure and we shall use dot product to find the angle in H – C – H.



Consider forces on the Carbon atom we know:

$$f_1 + f_2 + f_3 + f_4 = \mathbf{0} \text{ and } |f_1| = |f_2| = |f_3| = |f_4|$$

$$f_1 \bullet (f_1 + f_2 + f_3 + f_4) = f_1 \bullet \mathbf{0}$$

$$|f_1|^2 + |f_1||f_2|\cos\theta + |f_1||f_3|\cos\theta + |f_1||f_4|\cos\theta = 0$$

$$\cos\theta = -\frac{1}{3}$$

$$\theta = 109.4712206^0$$

In summary, the angles in CO_2, BF_3 and CH_4 are $\cos^{-1}\left(-\frac{1}{1}\right)$, $\cos^{-1}\left(-\frac{1}{2}\right)$ and $\cos^{-1}\left(-\frac{1}{3}\right)$ respectively.

Our next question is: Can we apply similar calculation to calculate the angle Cl – P – Cl in PCl_5 and get $\cos^{-1}\left(-\frac{1}{4}\right)$? If you search online, the angles for Cl – P – Cl in PCl_5 are 90°, 120° and 180°.

So, how can we reconcile this?

My take is try and imagine if CH_4 is forced to exist in 2-Dimension, what do you think is the angle for H – C – H? My bold guess is that if PCl_5 exists in 4-Dimension world then the angle Cl – P – Cl in PCl_5 will be $\cos^{-1}\left(-\frac{1}{4}\right)$.

Next question is: Even the angles for Cl – P – Cl in PCl_5 are 90°, 120° and 180°, which Cl is where to account for the different angles?

I wonder can it be that all Cl have no fixed position, every Cl is random and at every instance they can interchange with each other? I shall leave this to Chemistry teacher to explain.